TRINITY COLLEGE

Mathematics Specialist
Test 5 2018

Section 1 Calculator Free
Implicit Differentiation, Differential Equations

STUDENT'S NAME ____

DATE: Friday 10 August **TIME:** 20 minutes **MARKS**: 18

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

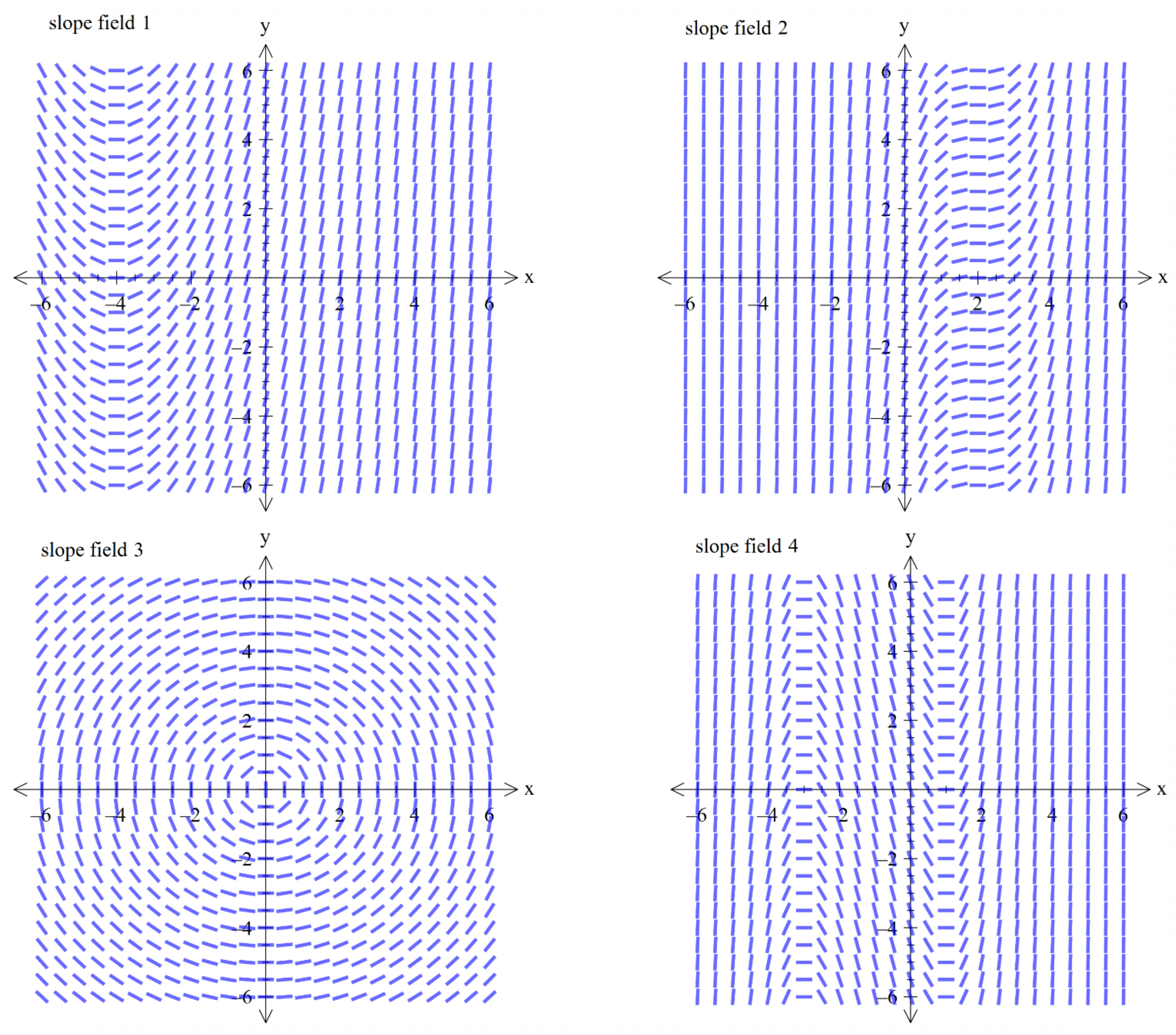
Solve the differential equation $\frac{dy}{dx} = \frac{-0.5x^2}{y}$ given $x = 0$ when $y = 2$.

2. (8 marks)

(a) From the seven differential equations given below, match four of them with the slope fields drawn. Enter results in the table below. [4]

A: $y' = x + 4$ B: $y' = -\dfrac{x}{y}$ C: $y' = \sqrt{x}$ D: $y' = (x+1)(x-3)$

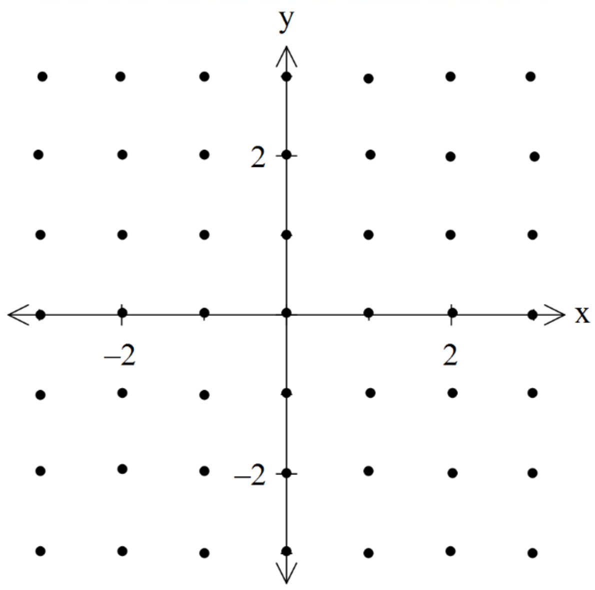
E: $y' = (x+3)(x-1)$ F: $y' = (x-2)^2$ G: $y' = \dfrac{x}{y}$



Slope field	1	2	3	4
Equation				

(b) For the differential equation $\frac{dy}{dx} = -2$

(i) Sketch the slope field [2]



(ii) Use your slope field to sketch a particular solution through the point (1,-3) [2]

3. (6 marks)

Solve the differential equation $\frac{dp}{dq} = 2pq(p+3)$ to give a general solution.

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Section 2 Calculator Assumed
Implicit Differentiation, Differential Equations

STUDENT'S NAME ____

DATE: Friday 10 August **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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4. (6 marks)

Elephant population on a reserve had been reduced by poaching to only 200 before a very strict anti-poaching policy allowed the elephants to recover. The population, P, increased according to the logistics model $\frac{dP}{dt} = 0.096P - 0.000016P^2$ where t is in years.

(a) Determine the maximum elephant population the reserve can sustain. [1]

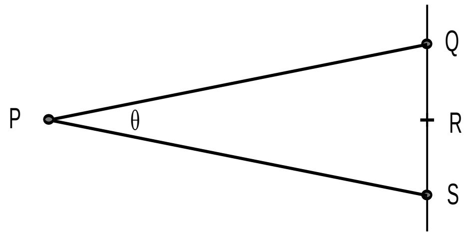
(b) Write an equation for the elephant population. [2]

(c) What is the rate of increase of the elephant population when the population reaches 1000? [1]

(d) How long will it take to reach 3000 elephants? [2]

5. (6 marks)

The diagram shows a hockey player at P running directly towards R, the midpoint of QS, where Q and S are the goalposts spaced 3.66 m apart at one end of a hockey pitch. PR is perpendicular to QS and θ, the shooting angle, is the size of angle QPS.



If the player is running at a constant speed of 6 m/s towards R, at what rate is the shooting angle θ increasing at the instant when the player is 9 m from R?

6. (9 marks)

Two variable resistors with resistance M Ohms and N Ohms respectively are connected in parallel so that the Total Resistance R Ohms is given by $\frac{1}{R} = \frac{1}{M} + \frac{1}{N}$.

(a) Use implicit differentiation to write a differential equation linking $\frac{dR}{dt}$, $\frac{dM}{dt}$ and $\frac{dN}{dt}$ [2]

(b) At the instant when $M = 50$ Ohms and $N = 200$ Ohms, M is increasing at a rate of 10 Ohms per minute.

(i) Determine R at this instant. [1]

(ii) Use Calculus methods to determine the rate of change of N (in Ohms per minute), at this instant, if R is increasing at a rate of 5 Ohms per minute. Show clearly how you obtained your answer. [2]

(c) Given that $M = N^2$, use the increments formula to calculate the approximate change in R when N changes from 50 Ohms to 51 Ohms. [4]

7. (9 marks)

A chemist places a lump of metal, initially at a temperature of 24°C into a hot research oven. The rate of change of temperature of the metal can be modelled by $\frac{dT}{dt} = k(450 - T)$ where T is the temperature in °C, t minutes after being placed in the oven and k is a positive constant. After 20 seconds, the temperature of the metal bar has risen by 39°.

(a) Show all steps to turn the given differential equation into the formula for T in terms of t and state the value of k. [5]

(b) What is the expected temperature of the lump of metal after 5 minutes? [1]

(c) When the temperature of the lump of metal is within 5° of its maximum, the power supply to the oven is cut off and no further heating occurs. After how many minutes does this occur? [3]